

Deriving Euler's Method for Two Coupled Differential Equations

For a system of two coupled differential equations of the form

$$\frac{dx}{dt} = f(t, x, y)$$

$$\frac{dy}{dt} = g(t, x, y)$$

Euler's method is given the recursion

$$x_{n+1} = x_n + hf(t_n, x_n, y_n)$$

$$y_{n+1} = y_n + hg(t_n, x_n, y_n)$$

where h is the step size.

For the system

$$\frac{dx}{dt} = -y$$

$$\frac{dy}{dt} = x$$

the recursion reads

$$x_{n+1} = x_n - hy_n$$

$$y_{n+1} = y_n + hx_n$$